

1.1 A combined-cycle, natural gas, power plant has an efficiency of 52%. Natural gas has an energy density of 55,340 kJ/kg and about 77% of the fuel is carbon.

- What is the heat rate of this plant expressed as kJ/kWh and Btu/kWh?
- Find the emission rate of carbon (kg C/kWh) and carbon dioxide (kg CO₂/kWh). Compare those with the average coal plant emission rates found in [Example 1.1](#).

Data from Example 1.1 ... a pulverized plant with a heat rate of 10,340 Btu/kWh burning a typical U.S. coal with a carbon content of 24.5 kgC/GJ (1 GJ = 10⁹ J). About 15% of thermal losses are up the stack and the remaining 85% are taken away by cooling water.

$$\text{MASTER'S EQN 1.1 : HEAT RATE } \left(\frac{\text{BTU}}{\text{kWh}} \right) = \frac{3412 \frac{\text{Btu}}{\text{kWh}}}{\eta}$$

IS A BIT VAGUE DEFINING HEAT RATE: η

Heat rate is one measure of the efficiency of a generator or power plant that converts a fuel into heat and into electricity. The heat rate is the amount of energy used by an electrical generator or power plant to generate one kilowatt-hour (kWh) of electricity. The U.S. Energy Information Administration (EIA) expresses heat rates in British thermal units (Btu) per net kWh generated. Net generation is the amount of electricity a power plant (or generator) supplies to the power transmission line connected to the power plant. Net generation accounts for all the electricity that the power plant consumes to operate the generator(s) and other equipment, such as fuel feeding systems, boiler water pumps, cooling equipment, and pollution control devices.

To express the efficiency of a generator or power plant as a percentage, divide the equivalent Btu content of a kWh of electricity (3,412 Btu) by the heat rate. For example, if the heat rate is 10,500 Btu, the efficiency is 33%. If the heat rate is 7,500 Btu, the efficiency is 45%.

<https://www.eia.gov/tools/faqs/faq.php?id=107&t=3> accessed 24 Jan 18

$$1 \text{ W} = \frac{1 \text{ J}}{\text{s}}$$

FURTHER, THE ORIGINS OF $3412 \frac{\text{Btu}}{\text{kWh}}$ IS UNCLEAR

SO, CONSIDER THE POWER EXPRESSION $\frac{1 \text{ kJ}}{\text{s}} = 1 \text{ kW}$

WHAT IS ENERGY ACCUMULATED IN 1 HOUR W/ 1 kW STEADY POWER?

$$1 \text{ kWh} = \frac{1 \text{ kJ}}{\text{s}} \cdot 3600 \text{ s} = 3600 \text{ kJ}$$

$$1 \text{ Btu} = 1.055 \text{ kJ}$$

$$3600 \text{ kJ} \cdot \frac{1 \left[\frac{\text{Btu}}{1.055 \text{ kJ}} \right]}{1.055 \text{ kJ}} = 3412.3 \text{ Btu}$$

THE NO. ABOVE

$$\eta = \frac{\text{ENERGY OUT (ELECTRICAL)}}{\text{ENERGY IN (THERMAL)}}$$

$$\eta = \frac{1 \text{ KWh}}{X \text{ BTUs}} = \frac{3412 \text{ BTU}}{X \times \text{BTUs}} = \frac{3412 \text{ BTU/1 KWh}}{X \text{ BTU/1 KWh}}$$

\equiv HEAT RATE

$$\text{HEAT RATE } \left(\frac{\text{BTU}}{\text{KWh}} \right) = \frac{3412 \text{ BTU/KWh}}{\eta}$$

$$a) \text{ BTU: HEAT RATE} = \frac{3412}{.52} = \underline{\underline{6,561 \text{ BTU/KWh}}} \text{ ANS.}$$

$$\text{KWh: HEAT RATE} = (6561) \left(1.055 \frac{\text{KJ}}{\text{KWh}} \right) = \underline{\underline{6922 \frac{\text{KJ}}{\text{KWh}}}} \text{ ANS.}$$

$$b) \text{ CARBON EMISSION RATE} = \frac{6922 \text{ KJ/KWh} \cdot .77 \text{ kgC}}{55,340 \text{ KJ/kg} \text{ kg fuel}}$$

$$= \underline{\underline{.0963 \text{ kgC/KWh}}} \text{ ANS.}$$

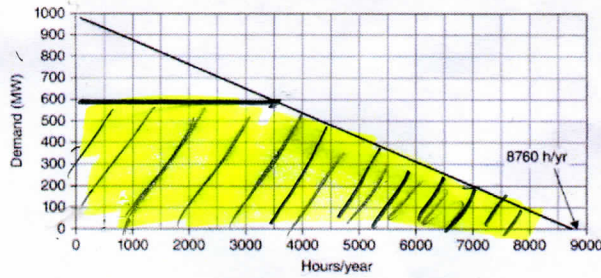
$$\text{CO}_2 \text{ EMISSION RATE} = .0963 \text{ kgC/KWh} \cdot \frac{44 \text{ kg CO}_2}{12 \text{ kgC}}$$

$$\text{COMPARISON:} = \underline{\underline{.353 \text{ kg CO}_2/\text{KWh}}} \text{ ANS.}$$

$$\frac{52\% \text{ EFFICIENT}}{33\% \text{ EFFICIENT}} = \frac{.0963 \text{ kgC/KWh}}{0.2673 \text{ kgC/KWh}} = .36$$

\Rightarrow 64% REDUCTION IN EMISSIONS
w/ MORE EFFICIENT PLANT

1.11 Consider the following very simplified load duration curve for a small utility.



- How many hours per year is the load less than 200 MW?
- How many hours per year is the load between 200 MW and 600 MW?
- If the utility has 600 MW of base-load coal plants, what would their average capacity factor be?
- Find the energy delivered by the coal plants.

$$a) \quad 8760 \text{ hrs} - 7000 \text{ hrs} = \underline{\underline{1760 \text{ hrs}}} \quad (P < 200 \text{ MW})$$

ANS.

$$b) \quad \underbrace{7000}_{200 \text{ MW}} - \underbrace{3500}_{600 \text{ MW}} = \underline{\underline{3500 \text{ hrs}}} \quad (200 < P < 600)$$

ANS.

$$c) \quad cf \triangleq \frac{\text{ENERGY PRODUCED}}{\text{ENERGY AT MAX}^{\text{CAPACITY}}$$

$$cf = \frac{600 \cdot 3500 + \frac{1}{2}(600)(8760 - 3500)}{600 \times 8760}$$

SHADED AREA UNDER CURVE

$$= .70$$

$$d) \quad W = .70 (600 \text{ MW})(8760 \text{ hrs})$$

$$= \underline{\underline{3.68 \times 10^6 \text{ MWh/yr}}} \quad \text{OR} \quad \underline{\underline{3.68 \frac{\text{TWh}}{\text{yr}}}} \quad \text{ANS.}$$

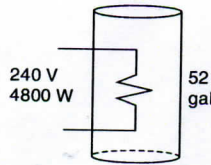


Figure P2.6

- 2.6. A 52-gallon electric water heater is designed to deliver 4800 W to an electric-resistance heating element in the tank when it is supplied with 240 V (it doesn't matter if this is ac or dc).
- What is the resistance of the heating element?
 - How many watts would be delivered if the element is supplied with 208 V instead of 240 V?
 - Neglecting any losses from the tank, how long would it take for 4800 W to heat the 52 gallons of water from 60°F to 120°F? The conversion between kilowatts of electricity and Btu/hr of heat is given by 3412 Btu/hr = 1 kW. Also, one Btu heats 1 lb of water by 1°F and 1 gallon of water weighs 8.34 lbs.
 - If electricity costs \$0.12/kWh, what is the cost of a 15-gal, 110°F shower if the cold-water supply is 60°F?

$$(a) \quad P = VI = I^2R = \frac{V^2}{R}$$

$$R = \frac{V^2}{P} = \frac{240^2}{4800} = \underline{12 \Omega} \text{ ANS}$$

$$(b) \quad P = \frac{(208)^2}{12} = \underline{3605 \text{ W}} \text{ ANS.}$$

$$(c) \quad +1 \text{ BTU} = +1^\circ \text{F} (1 \text{ LB})$$

$$[52 \text{ GAL H}_2\text{O}] \left[\frac{8.34 \text{ LB}}{\text{GAL}} \right] = 433.68 \text{ LBS}$$

$$\Rightarrow 433.68 \text{ BTU INCREASES 52 GAL } 1^\circ$$

$$\Rightarrow (60)(433.68) \text{ INCREASES 52 GAL } 60^\circ$$

$$\Rightarrow 26,020 \text{ BTU } \left. \vphantom{\Rightarrow} \right\} \text{ ENERGY REQUIRED.}$$

$$\text{SOURCE IS } 4.8 \text{ KW} \Rightarrow 4.8 (3412) \frac{\text{BTU}}{\text{HR}} = 16377.6 \frac{\text{BTU}}{\text{HR}}$$

$$t = \frac{26020 \text{ BTU}}{16377.6 \left(\frac{\text{BTU}}{\text{HR}} \right)} = \underline{1.588 \text{ HR}} \text{ TO HEAT 52 GALS } 60^\circ$$

$$(d) \quad 15 \text{ GAL} \left(\frac{8.34 \text{ LB}}{\text{GAL}} \right) (50^\circ) = 6322.5 \text{ BTU } \left. \vphantom{15 \text{ GAL}} \right\} \text{ ENERGY CHANGE}$$

$$t = \frac{6322.5 \text{ BTU}}{16377 \left(\frac{\text{BTU}}{\text{HR}} \right)} = .3867 \text{ HR}; \quad \$ = .3867 \text{ HR} (4.8 \text{ KW}) (.12 / \text{kWhr})$$

$$= \underline{\$.2233} \text{ ANS.}$$

2.7. Suppose an automobile battery is modeled as an ideal 12-V battery in series with an internal resistance of 0.01Ω as shown in (a) below.

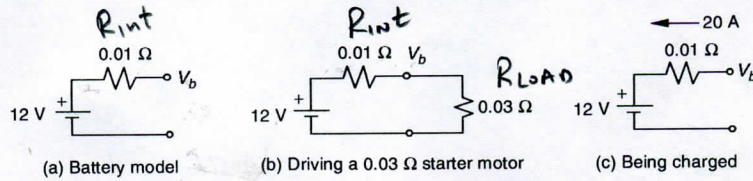


Figure P2.7

- What current will be delivered when the battery powers a 0.03Ω starter motor, as in (b)? What will the battery output voltage be?
- Compare the power delivered by the battery to the starter with the power lost in the battery's internal resistance. What percentage is lost in the internal resistance?
- To recharge the battery, what voltage must be applied to the battery in order to deliver a 20 A charging current as in (c)?
- Suppose the battery needs another 480 Wh of energy to be fully charged, which could be achieved with a quick charge of 80 A for .5h ($80A \times .5h \times 12V = 480Wh$) or a trickle charge of 10 A for 4 h. Compare Wh of energy lost in the internal resistance of the battery for each charging scheme.
- Automobile batteries are often rated in terms of their cold-cranking amperes (CCA), which is the number of amperes they can provide for 30 s at 0° F while maintaining an output voltage of at least 1.2 V per cell (7.2V for a 12-V battery). What would the CCA for the above battery (assuming the idealized 12-V source still holds)?

(a)
$$I = \frac{12}{0.01 + 0.03} = \underline{300 \text{ A}} \text{ ANS}$$

$$V_b = 12V - I R_{int} = 12 - 300(.01) = \underline{9V} \text{ ANS}$$

(b)
$$P_{LOAD} = I^2 R = (300)^2 (.03) = 2700W$$

$$P_{LOSS} = I^2 R = (300)^2 (.01) = 900W$$

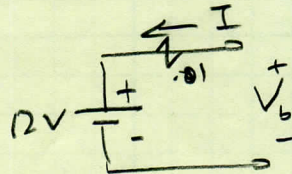
 ← 1800W DIFFERENCE

$$\% P_{LOST} = \frac{P_{LOSS}}{P_{LOSS} + P_{LOAD}} = \frac{900}{900 + 2700} = .25$$

(c)
$$V_b = I R_{int} + 12V$$

$$= 20(.01) + 12V$$

$$= \underline{12.2V} \text{ ANS}$$



(d) QUICK CHARGE 480Wh

$$P_{int} = I^2 R_{int}$$

$$= (80)^2 (.01)$$

$$= 64W$$

$$W = 64W (.5HR) = \underline{32Whr}$$

TRICKLE CHARGE

$$P_{int} = I^2 R_{int} = (10)^2 (.01)$$

$$= 1W$$

$$W = 1W (4HR) = \underline{4Whr}$$

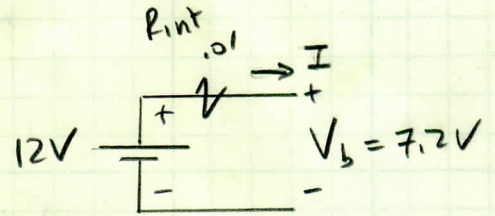
FAST CHARGE HAS 8X THE LOSS ANS.

(e)

$$12 = IR_{int} + V_b$$

$$= I(.101) + 7.2$$

$$I = \frac{4.8}{.101} = \underline{\underline{480 \text{ A}}} \text{ ANS. } \sim \text{CCA FOR BATTERY}$$



2.8 A photovoltaic (PV) system is delivering 15 A of current through 12-gage wire to a battery 80 ft away.

a. Find the voltage drop in the wires.

b. What fraction of the power delivered by the PVs is lost in the connecting wires?

c. Using Table 2.3 as a guide, what wire size would be needed to keep wire losses to less than 5% of the PV power output? (Assume the PVs will continue to keep the current at 15 A, which by the way, is realistic).

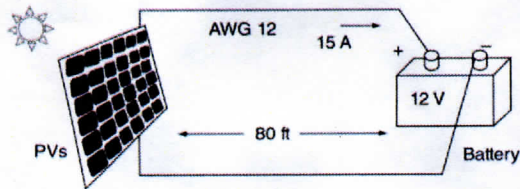


FIGURE P2.8

Wire Gage (AWG No.)	Diameter (in)	Area (cmils)	Ohms per 100 ft ^a	Max Current (A)
000	0.4096	168,000	0.0062	195
00	0.3648	133,000	0.0078	165
0	0.3249	106,000	0.0098	125
2	0.2576	66,400	0.0156	95
4	0.2043	41,700	0.0249	70
6	0.1620	26,300	0.0395	55
8	0.1285	16,500	0.0628	40
10	0.1019	10,400	0.0999	30
12	0.0808	6530	0.1588	20
14	0.0641	4110	0.2525	15

TABLE 2.3 Characteristics of Copper Wire

a) PER TABLE 2.3, 12-ga WIRE'S $R = .1588 / 100'$
 FOR 160' OF WIRE

$$\Delta V = RI = 15A \times 160' \times \frac{.1588 \Omega}{100'} = 15 \cdot .254 \Omega$$

$$= \underline{\underline{3.81V}} \text{ ANS}$$

b) $P_{PV} = IV = 15(12 + 3.81)V = 237W$

$$P_{loss} = I^2 R = (15)^2 (.254) = 57.2W$$

$$\% \text{ LOSS IN WIRE} = \frac{57.2}{237} = 24\%$$

c) 5% PV-POWER = P_R

$$.05 I (12V + IR) = I^2 R$$

$$.05 (15) (12 + 15R) = 15^2 R$$

$$.6 + .75R = 15R$$

$$R = \frac{.6}{14.25} = \frac{.0421 \Omega}{160'} \Rightarrow R \leq \frac{100}{160} \cdot .0421 = \frac{.026 \Omega}{100'}$$

4-ga WIRE IS .0249 Ω / 100'
ANS.

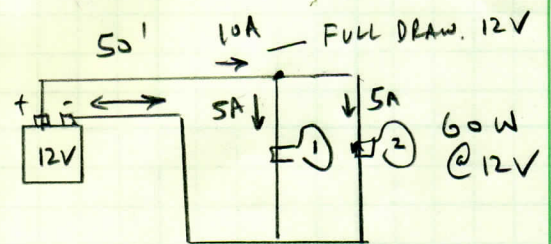
- 2.9. Consider the problem of using a low-voltage system to power a small cabin. Suppose a 12-V system powers a pair of 60-W lightbulbs (wired in parallel). The distance between these loads and the battery pack is 50 ft.
- Since these bulbs are designed to use 60 W at 12 V, what would be the filament resistance of each bulb?
 - What would be the current drawn by two such bulbs if each receives a full 12 V?
 - Of the gages shown in Table 2.3, what gage wire should be used if it is the minimum size that will carry the current?
 - Find the equivalent resistance of the two bulbs plus the wire resistance to and from the battery. Both lamps are turned on (in this and subsequent parts).
 - Find the current delivered by the battery with both bulbs turned on.
 - Find the power delivered by the battery.
 - Find the power lost in the connecting wires in watts and as a percentage of battery power.
 - Find the power delivered to the bulbs in watts and as a percentage of their rated power.

$$P = \frac{V^2}{R}$$

$$P = VI =$$

$$(a) \quad P = VI = \frac{V^2}{R} = \frac{(12)^2}{R}$$

$$R = \frac{(12)^2}{60} = \frac{144}{60} = \underline{2.4 \Omega} \text{ ANS.}$$



$$(b) \quad P = I^2 R \Rightarrow I^2 = \frac{P}{R} = \frac{60}{2.4} \Rightarrow I = \sqrt{\frac{60}{2.4}} = \underline{5 A} \text{ ANS}$$

(c) 10 A TO SUPPLY BOTH BULBS @ 5 A

\Rightarrow 14 GAGE IS MIN SIZE w/ 15 A MAX CURRENT. ANS.

$$(d) \quad R_{\text{WIRE}} = 50' \times 2 \times \frac{.2525}{100'} \quad R_{\text{BULBS}} = (2.4 \Omega) / (2.4 \Omega)$$

$$= .2525 \Omega \quad = 1.2 \Omega$$

$$R_{\text{TOT}} = R_{\text{WIRE}} + R_{\text{BULBS}} = .2525 + 1.2$$

$$= \underline{1.4525 \Omega} \text{ ANS}$$

$$(e) \quad I = \frac{V}{R_{\text{TOT}}} = \frac{12}{1.4525} = \underline{8.26 A} \text{ ANS.}$$

$$(f) \quad P_{\text{BAT}} = 12 (8.26) = \underline{99.14 W} \text{ ANS.}$$

$$(g) \quad P_{\text{WIRE LOSS}} = I^2 R_{\text{WIRE}} = (8.26)^2 (.2525) = \underline{17.22 W} \text{ ANS.}$$

$$\% P_{\text{WIRE LOSS}} = 17.2$$

$$(h) \% P_{\text{WIRE LOSS}} = \frac{17.22 \text{ W}}{99.14 \text{ W}} = 17.36\%$$

$$(h) I_{\text{BULB}} = \frac{8.26 \text{ A}}{2} = 4.13 \text{ A}$$

$$P_{\text{BULB}} = I_{\text{BULB}}^2 R_{\text{BULB}} = (4.13)^2 (2.4) = \underline{40.94 \text{ W}} \text{ ANS} \quad \text{1 BULB}$$

$$P_{\text{BULBS}} = 2 P_{\text{BULB}} = \underline{81.88 \text{ W}} \text{ ANS} \quad \text{2 BULBS}$$

$$\% P_{\text{BULBS}} = \frac{81.88}{99.14 \text{ W}} = \underline{82.5\%} \text{ ANS.}$$